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Quantum Corrections to Hawking Radiation for Rainbow Universe

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Abstract Via the method beyond semi-classical approximation, we obtain the correctional tunneling probability and Hawking temperature at the apparent horizon of Finsler rainbow universe. Then we apply Bekenstein–Hawking entropy area law used in black hole to the cases of rainbow universe, and reach the entropy of the apparent horizon. Finally, we calculate the correctional entropy and obtain reasonable results.

Keywords Beyond semi-classical approximation · Tunneling probability · Hawking temperature · Apparent horizon · Finsler rainbow universe

1 Introduction

Generally, Hawking radiation was thought as the property of black hole. However, the related Bekenstein–Hawking entropy and Hawking temperature at the apparent horizon of FRW universe have been presented in some references recently [1–3]. The aim of this paper is to study Hawking temperature at the apparent horizon of rainbow universe considered the effect of rainbow gravity. Since Hawking radiation was proposed, researchers have done some work about the interesting quantum effect. Recently, Kraus and Wilczek et al. put forward the tunneling principle to investigate Hawking radiation of black hole [4–29]: in the process of tunneling, the tunneling probability depends on the imaginary part of action; what is more, ingoing particles must be real. On the basis of this theory, we can get the imaginary part of radiated scalar particles' action with Klein–Gordon equation. Finally, the tunneling probability and Hawking temperature in curved space-time can be obtained. However, in calculating, researchers only care about the leading term of action expanded with regard to $\hbar$, so their corresponding results are semi-classical approximate.

Considering the limitation of semi-classical approximation, Banerjee and Majhi proposed the method beyond semi-classical approximation in order to solve the problems resulting from semi-classical approximation [30–40]. With this new method, they calculated

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all the terms of action expanded with regard to $\hbar$. Now not only the results in semi-classical approximation but the correctional terms beyond semi-classical approximation are covered. In the meantime we show that the appropriate choice of dimensionless parameter can reproduce one loop reaction effects which are the source of quantum corrections. Following this method, researchers have studied quantum tunneling at the horizon of series of space–times, and the results in these cases are consistent with ones from the theory of one loop quantum gravity. In this paper we will show the fact that this new method may do help to explain explicitly the phenomena from quantum gravity.

For a long time, researchers try to establish a complete quantum gravitational theory, which is hoped to contribute to study the early universe, and the high energy physics. Nowadays, Magueijo and Smolin have proposed the theory of rainbow gravity. In this theory, with equality principle in deformed general relativity requiring that the free falling observers with some energy can observe the same physics laws in space–time, the energy-momentum tensor and Einstein equation will change in curved space–time, and the deformed Lorentz relation is

$$l_1^2 E^2 - l_2^2 \vec{p} \bullet \vec{p} = m^2 \tag{1}$$

and the Einstein field equation becomes

$$G_{\mu\nu}(E) = 8\pi G(E) T_{\mu\nu}(E) + g_{\mu\nu}\Lambda(E). \tag{2}$$

Where, both Newton factor $G(E)$ and universe factor $\Lambda(E)$ depend on the energy of particles. Using deformed energy-momentum tensor and Einstein equation above, people have got the FRW metric as

$$ds^2 = -\frac{dt^2}{l_1^2} + \frac{a^2(t)}{l_2^2}\left[\frac{dr^2}{1-\kappa r^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]. \tag{3}$$

This is just the standard rainbow FRW universe metric. Where, l_1 and l_2 are energy dependent. So from the formalism above we can see that the structure of this space–time correlates with the energy of particles moving in this space–time, namely is tangent vector dependent. So far, people have done some initial research to this metric.

Recently, Cai et al. [41, 42] studied the Hawing radiation at apparent horizon of universe. In this paper, we will extend their work to study quantum tunneling from rainbow FRW universe using the method beyond semi-classical approximation mentioned above, and then discuss the Hawking radiation and Bekenstein–Hawking entropy at the apparent horizon of Finsler rainbow universe. The structure of this paper is organized as follows: in the second section, we will calculate the tunneling probability and Hawking temperature of rainbow universe, furthermore obtain their correctional terms to the semi-classical approximation. Some discussion about the correctional entropy at the apparent horizon will be concluded in the third section.

2 Scalar Particles Tunneling from Rainbow Universe

For observing, universe should be flat, and it means the constant k is vanishing. Following the method proposed by Cai [41], we apply the coordination transformation below

$$\tilde{r} = l_1 a(t) r \tag{4}$$

inserting (4) into (3), we have the FRW universe metric

$$ds^2=\left(-\frac{1}{l_1^2}+\frac{\tilde{r}^2H^2}{l_2^2l_1^2}\right)dt^2+\frac{d\tilde{r}^2}{l_2^2l_1^2}-2H\tilde{r}\frac{dtd\tilde{r}}{l_2^2l_1^2}+\frac{\tilde{r}^2}{l_2^2l_1^2}(d\theta^2+\sin^2\theta d\varphi^2). \tag{5}$$

Here, we introduce the Hubble factor $H(H=\dot{a}/a, \dot{a}=\partial a/\partial t)$. In this space–time, the null hypersurface equation complied by the metric above is

$$g^{\mu\nu}\partial_\mu\tilde{r}\partial_\nu\tilde{r}=0. \tag{6}$$

So the apparent horizon of this universe is $\tilde{r}_A=l_2H^{-1}$. Next we will investigate the quantum tunneling at apparent horizon of rainbow universe. Firstly the scalar particles equation described by Klein–Gordon equation is

$$\frac{-\hbar^2}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g}\partial_\nu)\phi+m^2\phi=0. \tag{7}$$

In order to solve this equation, we now come to a massless scalar field function ϕ, which satisfies the relationship above

$$\phi\sim e^{-\frac{i}{\hbar}S(t,\tilde{r},\theta,\varphi)}. \tag{8}$$

Then, we put (8) into (7). However, the tunneling trajectory is radical, so only the $(\tilde{r},t)$ section is important in our calculation. The final Klein–Gordon equation can be simplified as

$$\begin{aligned}&\left(\frac{\partial S}{\partial t}\right)^2+i\hbar\frac{\partial^2 S}{\partial t^2}+2H\tilde{r}\frac{\partial S}{\partial t}\frac{\partial S}{\partial\tilde{r}}+i\hbar H\tilde{r}\frac{\partial^2 S}{\partial\tilde{r}\partial t}+i\hbar H\frac{\partial S}{\partial t}+i\hbar H\tilde{r}\frac{\partial^2 S}{\partial t\partial\tilde{r}}\\&\quad+2i\hbar H^2\tilde{r}\frac{\partial S}{\partial\tilde{r}}-(l_2^2-H^2\tilde{r}^2)\left(\frac{\partial S}{\partial\tilde{r}}\right)^2-i\hbar(l_2^2-H^2\tilde{r}^2)\frac{\partial^2 S}{\partial\tilde{r}^2}=0\end{aligned} \tag{9}$$

where, $S(\tilde{r},t)$stands for the action of particles. Through WKB approximation, we expand it in powers of $\hbar$. So this action has

$$S(\tilde{r},t)=S_0(\tilde{r},t)+\sum_i\hbar^iS_i(\tilde{r},t). \tag{10}$$

In the equation above, S_0 is the semi-classical part of action S. Inserting (10) into (9), we separate the Klein–Gordon equation as regard to different powers of $\hbar$, we can have following set of functions.

$$\hbar^0:\quad \left(\frac{\partial S_0}{\partial t}\right)^2+2H\tilde{r}\frac{\partial S_0}{\partial t}\frac{\partial S_0}{\partial\tilde{r}}-(l_2^2-H^2\tilde{r}^2)\left(\frac{\partial S_0}{\partial\tilde{r}}\right)^2=0 \tag{11}$$

$$\hbar^1:\quad \frac{\partial S_0}{\partial t}\frac{\partial S_1}{\partial t}+H\tilde{r}\frac{\partial S_0}{\partial t}\frac{\partial S_1}{\partial\tilde{r}}+H\tilde{r}\frac{\partial S_1}{\partial t}\frac{\partial S_0}{\partial\tilde{r}}-(l_2^2-H^2\tilde{r}^2)\frac{\partial S_0}{\partial\tilde{r}}\frac{\partial S_1}{\partial\tilde{r}}=0 \tag{12}$$

$$\ldots$$

Decomposing (11), we get

$$\frac{\partial S_0}{\partial t}=(-H\tilde{r}\pm l_2)\frac{\partial S_0}{\partial\tilde{r}}. \tag{13}$$

We also decompose (12), then make use of (13). There is

$$\frac{\partial S_1}{\partial t} = (-H\tilde{r} \pm l_2)\frac{\partial S_1}{\partial \tilde{r}}. \tag{14}$$

Similarly, going on the step above we can get all relationships about S_i with regard to higher powers of $\hbar$. Then we are surprised to find that all the relationships are not independent. Any S_i is always proportional to S_0. So action $S(\tilde{r},t)$ can be rewritten as

$$S(\tilde{r},t) = \left(1 + \sum_{i=1} \beta^i \hbar^i\right) S_0(\tilde{r},t). \tag{15}$$

To get the value of action S, we must firstly compute S_0. Because S_0 is both radius and time dependent, but the radius and time are independent. So we can define the deviation to time of the semi-classical approximate action S_0 as

$$\frac{\partial S_0}{\partial t} = -E. \tag{16}$$

Then, plugging (16) into (13), we have

$$S_0(\tilde{r},t) = -\int E dt + \int \frac{-1}{-H\tilde{r} \pm l_2} E d\tilde{r}. \tag{17}$$

And inserting (17) into (15), we reach

$$S(\tilde{r},t) = \left(1 + \sum_{i=1} \beta^i \hbar^i\right)\left(-\int E dt + \int \frac{-1}{-H\tilde{r} \pm l_2} E d\tilde{r}\right) \tag{18}$$

where, the sign $+/-$ stands for outgoing and ingoing solution of action respectively. From the equation above we can see that the action covers the correctional terms to the semi-classical approximation. Given the action, we can get ingoing and outgoing solution of scalar particles' wave function respectively

$$\phi_{in} = \exp\left[-\frac{i}{\hbar}\left(1 + \sum_{i=1} \beta^i \hbar^i\right)\left(-\int E dt + \int \frac{-1}{-H\tilde{r} + l_2} E d\tilde{r}\right)\right] \tag{19}$$

$$\phi_{out} = \exp\left[-\frac{i}{\hbar}\left(1 + \sum_{i=1} \beta^i \hbar^i\right)\left(-\int E dt + \int \frac{-1}{-H\tilde{r} - l_2} E d\tilde{r}\right)\right]. \tag{20}$$

The Hawking radiation is radical behavior, so in (18) only the integration of action with regard to radius is related. Their imaginary parts are

$$\mathrm{Im} S_{out} = \left(1 + \sum_{i=1} \beta^i \hbar^i\right)\left(\int \frac{-1}{-H\tilde{r} - l_2} E d\tilde{r}\right) \tag{21}$$

$$\mathrm{Im} S_{in} = \left(1 + \sum_{i=1} \beta^i \hbar^i\right)\left(\int \frac{-1}{-H\tilde{r} + l_2} E d\tilde{r}\right). \tag{22}$$

Integrating the two equations above at the apparent horizon, so the values of their imaginary parts are

$$\mathrm{Im} S_{out} = 0, \qquad \mathrm{Im} S_{in} = \left(1 + \sum_{i=1} \beta^i \hbar^i\right)\pi \frac{E\tilde{r}_A}{l_2}. \tag{23}$$

From (23), the whole imaginary part of action is

$$\mathrm{Im} S = \mathrm{Im} S_{out} - \mathrm{Im} S_{in} = -\left(1 + \sum_{i=1} \beta^i \hbar^i\right)\pi \frac{E\tilde{r}_A}{l_2}. \tag{24}$$

Meanwhile, the outgoing and ingoing wave probabilities of radiated particles are, respectively

$$P_{out} = |\phi_{out}|^2, \qquad P_{in} = |\phi_{in}|^2. \tag{25}$$

It is showed that the time part of action does temporal contribution to the outgoing and ingoing wave probabilities of radiated particles. Now, using the principle of detailed balance [43–46], we obtain the tunneling probability as

$$\Gamma = \frac{P_{in}}{P_{out}} \sim \exp(-2\mathrm{Im} S) = \exp\left[\left(1 + \sum_{i=1} \beta^i \hbar^i\right)\pi \frac{2E\tilde{r}_A}{l_2}\right]. \tag{26}$$

And Hawking temperature is

$$T_A = \frac{l_2}{2\pi\tilde{r}_A}\left(1 + \sum_{i=1} \beta^i \hbar^i\right)^{-1} = T_0\left(1 + \sum_{i=1} \beta^i \hbar^i\right)^{-1}, \tag{27}$$

where $T_0 = l_2(2\pi\tilde{r}_A)^{-1}$ is semi-classical Hawking temperature. The latter part in the equation above is quantum correction, in which there is an undetermined parameter β^i that can be determined by the proportional relationship between S_o and S_i. Since S_o has dimension of $\hbar$, parameter β^i should have dimension of $\hbar^{-i}$. In the unit in which $\hbar = c = 1$, the Plank constant is order of Plank length. So parameter β^i has dimension of $\tilde{r}_A^{-2i}$, namely β^i amounts to $\tilde{r}_A^{-2i}$ (α_i is dimensionless constant). Explicitly, both tunneling probability and Hawking temperature are energy dependent. From now on, we would recognize rainbow universe more clearly.

3 The Entropy of the Apparent Horizon

Applying the semi-classical Bekenstein–Hawking area law for black hole into rainbow universe, we can take the entropy at the apparent horizon of spherically symmetrical rainbow universe as

$$S_{BHA} = \frac{A}{4\hbar}. \tag{28}$$

Here, A is the area of the apparent horizon. Taking the correctional Hawking temperature into account, we have the entropy change relationship

$$dS_{bha} = \frac{dE}{T_A}. \tag{29}$$

Putting (27) into (29), then integrating , we have

$$S_{bha} = \int \frac{1}{T_0}\left(1 + \sum_{i=1} \beta^i \hbar^i\right) dE = \frac{A}{4\hbar} + \frac{\pi\alpha_1}{l_2}\ln A + const + \cdots. \tag{30}$$

We can notice that the first term is the semi-classical entropy, others are the quantum corrections. We need to note that the semi-classical approximate entropy change is

$$dS_{BHA} = \frac{dE}{T_0} \tag{31}$$

and the entropy beyond semi-classical approximation is the formula described by (29). In this paper, we apply the Hamilton–Jacobi method beyond semi-classical approximation usually used in black hole to study scalar particles tunneling from rainbow universe. Through taking all the terms into account, we compute action of particles. Then we obtain the tunneling probability and Hawking temperature at the apparent horizon. Furthermore, we recognize the imaginary part of action's outgoing solution does no contribution. And both tunneling probability and Hawking temperature correlate with l_2 and l_1 which are energy dependent. This phenomenon is no other than the results of rainbow gravity effects. Mathematically, this means that the structure and features of rainbow universe are related to tangent vector. At this time, we can see that Riemannian geometry is no longer perfect in describing the characters of rainbow universe. So we are in urgent need of a new geometry—Finsler geometry to overcome this difficult. Resulting from this reason, some persons have pointed out that the explaining of rainbow gravitational theory lies on Finsler geometry [47–50]. In this aspect, some achievements have been got. On the other hand, in this paper (4) is so important that we may have different solutions in other coordination transformations. Certainly, in some circumstances, we should care about the energy varying from time to time, and all of this need further study.

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